akuTEK

www.akutek.info

PRESENTS

Schroeder Frequency Revisited

by Magne Skålevik

This is a preprint of a paper presented at Forum Acusticum 2011

ABSTRACT

In the middle of the 20th century, Manfred Schroeder explored the transition region of the room acoustical frequency response, namely the cross-over between the low frequency region dominated by separate modes and the high frequency region dominated by dense modal overlap with statistical (Gaussian) properties. The cross-over region is not an abrupt one, and the definition of a limiting frequency is not obvious. Indeed, Schroeder first suggested a theoretical average ten-fold modal overlap as a criterion for the high frequency region, but after years of experience with measurement results, he found it more proper to require only 3-fold modal overlap (Schroeder 1962). This led to the famous Schroeder Frequency, $Fs=2000\cdot(T/V)^{0.5}$

However, it can be deduced from Schroder's theory that from measurements one cannot with certainty detect a lower limit for the high frequency region that is higher than approximately $0.5\cdot Fs$. Besides, one can in many small room measurements even below $0.5\cdot Fs$ find statistical properties of the frequency response which is indistinguishable from the high frequency region. Since the cross-over frequency has so many important implications in room acoustics, it is worth while having a closer look at the cross-over region itself. This is indeed the objective of this paper. A new limiting frequency will be suggested.

akuTEK navigation:

Home
Papers
Articles
Title Index
akuTEK Research
Stochastics (parts 1 and 2)

Forum Acusticum 2011 - Aalborg - Denmark

Schroeder Frequency Revisited

Magne Skålevik
AKUTEK www.akutek.info
Senior acoustical consultant, Brekke & Strand akustikk, Oslo, Norway

Summary
In many smaller rooms, existing or under planning, it is important to be able to determine the high frequency region in which Schroeder‟s theory applies. From the discussion in this paper, it is concluded that it remains to determine a measurable cross-over frequency or cross-over region between the high frequency Schroeder region and the lower frequency Modal Region. Several possible candidates are presented, together with suggested criteria for verification or falsification.

1. Abstract

In the middle of the 20th century, Manfred Schroeder explored the transition region of the room acoustical frequency response, namely the cross-over between the low frequency region dominated by separate modes and the high frequency region dominated by dense modal overlap with statistical (Gaussian) properties. The cross-over region is not an abrupt one, and the definition of a limiting frequency is not obvious. Indeed, Schroeder first suggested a theoretical average ten-fold modal overlap as a criterion for the high frequency region, but after years of experience with measurement results, he found it more proper to require only 3-fold modal overlap (Schroeder 1962). This led to the famous Schroeder Frequency, $Fs=2000\cdot(T/V)^{0.5}$
However, it can be deduced from Schroder's theory that from measurements one cannot with certainty detect a lower limit for the high frequency region that is higher than approximately $0.5\cdot Fs$. Besides, one can in many small room measurements even below $0.5\cdot Fs$ find statistical properties of the frequency response which is indistinguishable from the high frequency region. Since the cross-over frequency has so many important implications in room acoustics, it is worth while having a closer look at the cross-over region itself. This is indeed the objective of this paper. A new limiting frequency will be suggested.

2. Background

In his paper “On frequency response curves in rooms”[4] , Manfred Schroeder wrote (1962):

“Since Wente[1] in 1935 directed the attention of room acousticians to stationary frequency response curves of rooms, a number of papers have been devoted to this topic. A certain completion of these investigations was reached in 1954 in papers by Schroeder[3], Kuttruff and Thiele[5], who were able to show both theoretically and experimentally that above a certain critical frequency the statistical parameters of frequency response curves for all rooms are either identical or depend at most on reverberation time. Specifically the average frequency spacing (Figure 1) between adjacent maxima was found to be $<Fmax> = 6.7/T_{60}$, where T_{60} is the reverberation time for a 60-dB decay. The above-mentioned critical frequency is given by

$$F_c \sim 2000\cdot(T_{60}/V)^{0.5} \qquad (1)$$

where V is the volume of the room in cubic meters. Recently, a new theory for $<\Delta Fmax>$ has been developed which is considerably simpler and uses fewer assumptions than the earlier theory. Its main result which is derived (…) is $<\Delta Fmax> = (3.91\pm0.04)/T_{60}$”

Since the experiments involved counting the maxima and of the frequency level curves, level resolution and frequency resolution of the measurement technique was critical, as discussed by Schroeder in the 1962 paper[3]. For example, by measuring levels in 0.5dB steps one would reveal more maxima than with 1.0dB steps.

Running Monte Carlo computations, letting the dB-quantisation approaching zero, came out with < ΔFmax > decreasingly approaching $3.90/T_{60}$, in close agreement with the theoretical result above.

Also, the frequency resolution was an important issue: The loudspeaker used for the excitation of the room was supplied with a sinusoidal voltage whose frequency was changed slowly enough to ensure stationary condition[2] (otherwise one might loose details in the frequency response curve).

One application in this newly explored field of knowledge was to control feedback in loudspeaker systems: "Improvement of acoustic feedback stability by frequency shifting"[6] (1964).

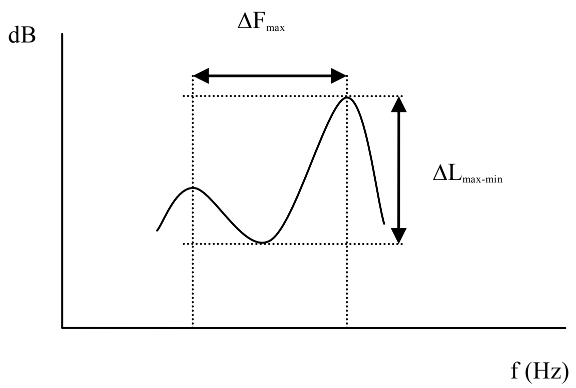

Figure 1 Examples of frequency spacing between maxima, ΔF_{max} , and level difference of fluctuations between maxima and minima, $\Delta L_{max\text{-}min}$, of the frequency response in a room.

Revisiting his own Schroeder frequency (1996), Manfred Schroeder ignores the decimals and the subscript "60" behind the T, expressing the average frequency spacing between maxima as follows:

$$< \Delta Fmax > = 4/T \qquad (2)$$

This simple expression contains the only variable statistical parameter in high frequency (HF) room acoustics. Level statistics (standard deviation, percentiles, etc) of the frequency responses (FRs) in the HF region are pure lottery outcomes, and will be the same in any room when variations in average FR is neglected.

Schroeder"s own words: "...the sound transfer function above Fc between two „distant"" points (i.e. points with negligible direct power transmission) can be considered an approximate complex Gaussian process with a number of well known consequences (e.g. average frequency spacing between relative maxima of the power transfer function equal to 4/T, average range of the statistical fluctuations between maxima and minima equal to 10 dB, etc.).", see Figure 1.

So, at sufficient distance from a source, high-frequency acoustics of any room can be characterised by (A) the 4/T dependent maxima spacing, and (B) the room-independent level statistics.

3. The lower limit of the high frequency region

Above, we have only referred to "some critical" frequency, above which high frequency room acoustic characteristics apply. How did Schroeder actually arrive at the expression in (1), and in particular, why the apparently accurate factor of 2000 multiplying the square root of the ratio of the room volume to the reverberation time?

Even though Schroeder in his 1996 paper refers to the Schroeder Frequency as a cross-over between the low-frequency region and the high frequency region, his justification for (1) was that by 1962, measurements by various authors had shown that the high-frequency theory is actually valid for frequencies as low as $2000\cdot(T/V)^{0.5}$. In his 1954 paper he proposed the factor 4000, which he in a comment in his 1962 paper considered "more conservative". Obviously, the intention was to be certain that the limit was set high enough in order to maintain validity of the high-frequency theory.

By modal theory, the high frequency region will have a high degree of modal overlap where half-power bandwidths of the modes are large compared to the average spacing of modes.

Half-power bandwith B depends only on reverberation time

$$B = \frac{\log_e 10^6}{2\pi T} = \frac{2.2}{T} \qquad (3)$$

And the average spacing Δf between modes in any frequency region can be calculated from

$$\Delta f = \frac{c^3}{4\pi V f^2} \qquad (4)$$

where c is the speed of sound in m/s, V is the room volume in m^3, and f is the center frequency of the region in Hz.

In 1954, Schroeder suggested a 10-fold overlap of modes, equivalent to requiring $B=10\cdot\Delta f$, as a low limit for the high frequency region. Combining (3) and (4) then leads to a low limit equal to $f_c=4000\cdot(T/V)^{0.5}$. As mentioned above, this was a conservative arbitrary choice, which eight years later, motivated by more measurement data, was succeeded by another arbitrary choice, namely $f_c=2000\cdot(T/V)^{0.5}$, corresponding to a 3-fold overlap.

Summing up the above, the Schroeder Frequency is designed and tested as a low limit ensuring the validity of high frequency theory. So, while the limit is set sufficiently high, it may still be set higher than necessary.

For the purpose of this paper, we shall hereafter refer to the high-frequency region, i.e. were Schroeder‟s high-frequency theory is valid, as the Schroeder Region. Further we shall denote the low frequency region where average peak spacing obeys (4), the Modal Region.

4. Pursuing the measurable cross-over frequency

We require a cross-over frequency or at least a cross-over frequency region which separates the Schroeder Region from the low-frequency region, and which can be measured. The Schroeder Frequency was not intended to meet this requirement.

Though experimental data justifies that the low limit of the Schroeder Region can be set as low as the Schroeder Frequency, it is not yet justified that it cannot be set lower. On the contrary, there are examples that the statistical properties of the high region extends considerably lower than the Schroeder Frequency[8]. In the same article, this author discusses several alternative candidates for a cross-over frequency.

A major problem is the fact that the Schroeder Frequency $f_c=2000\cdot(T/V)^{0.5}$ can not be detected directly by measurements.

An obvious alternative approach is to look at the measurable characteristic properties of the two regions and try to determine the frequency regions in which such properties are observed.

According to Schroeder there are basically two properties that characterize the Schroeder Region, namely

1. The average spacing between maxima
2. The average level range of fluctuations between maxima and minima

Starting with the first characteristic property, one can both measure and calculate that in the Schroeder Region the average spacing between maxima is $<\Delta Fmax> = 4/T$. By contrast, in the very low end of the frequency scale, the observed maxima are the mode peaks. The spacing can never be more than 170/L in a room with length L, but toward higher frequencies, the average frequency spacing between observed maxima will decrease as $1/f^2$ according to (4), see Figure 2.

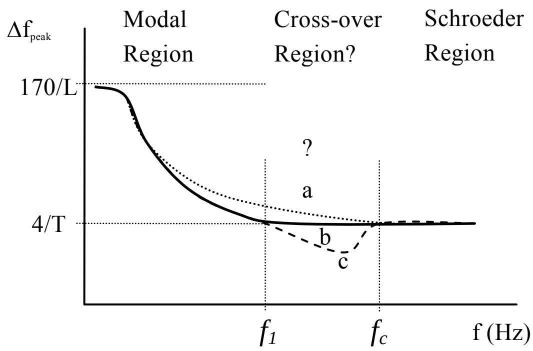

Figure 2 Average peak spacing approching 4/T towards Schroeder Region. Transition is not clear, alternatives a, b and c are suggested.

From (4) one can predict that around the frequency $f_1 \sim 900\cdot(T/V)^{0.5}$ the average spacing between <u>modes</u> will equal $\Delta f=4/T$, exactly the same as the spacing of maxima in the Schroeder Region. This frequency is slightly more than one octave below the Schroeder frequency, since $f_1 = 0.45\cdot f_c$. As a consequence, one cannot from a measurement of average peak spacing alone know wether the region about f_1 belongs to the Schroeder Region or to the Modal Region.

On the other hand, in any frequency region where the measured average peak spacings deviate from both (4) and 4/T, we can state that it belongs to neither the Modal Region nor the Schroeder Region. In other words, such an observation would be evidence of a transition region. In Figure 2, three alternative transitions (a), (b) and (c) from the Modal Region to the Schroeder Region are suggested.

(a) suggests a long transition region of deviation from (4) combined with f_c being the definite low limit of the Schroeder region. Such significant deviation from (4) is less likely to occur around f_1 from the fact that the average modal spacing is larger than the modal bandwidth for $f < 1.3 \cdot f_1$.

(c) like (a) suggest f_c being the definite low limit of the Schroeder region, but has a very short transition between the Modal Region and the Schroeder Region.

(b) represents the possibility that the frequency spacing of the Schroeder Region extends below f_c, and without any transition region enters directly the upper Modal Region at f_1.

Now, keep in mind the second characteristic property of the Schroeder region stated above. If the level statistics, in particular if the measured average level range of fluctuations between maxima and minima are observed to deviate from that of the Schroeder Region, one will have evidence of being below the Schroeder Region, regardless of frequency spacing of peaks.

Several measurements have proved that level distribution remains unchanged also somewhat below f_c. If this turns out to be the general case, then the frequency spacing will again be the critical parameter.

Note: There are textbooks interpreting the Schroeder Frequency as the lower limit for a cross-over region that spans all of two octaves from f_c to $4 \cdot f_c$. However, this author has not found justification this in measurement or theory, and does not seem to be compatible with the results of Schroeder's work.

5. Conclusion

From the discussion above, it is concluded that it remains to determine a measurable cross-over frequency or cross-over region between the high frequency Schroeder region and the lower frequency Modal Region. Several possible candidates are presented, together with suggested criteria for verification or falsification.

6. Further work

In further work, the possible limits and transitions suggested above will be investigated in measured frequency responses of rooms with Schroeder Frequencies well above 100Hz (not large rooms) in order to have a wide enough Modal Region, and 170/L larger than 4/T.

References

[1] E. C. Wente, J. Acoust. Soc. Am. 7, 123 (1935)

[2] R . H. Bolt and R . W. Roop,J . AcoustS. oe.A m. 22, 280 (1950)

[3] M. R. Schroeder: Die statistischen Parameter der Frequenzkurven von grossen Räumen. Acustica 4 (1954) 594-600. Note: In his original 1954 paper he proposed a more „conservative" factor of 4000 instead of 2000 in Eq. 1, corresponding to a tenfold modal overlap.

[4] M. R. Schroeder and K. H. Kuttruff, „On Frequency Response Curves in Rooms. Comparison of Experimental, Theoretical, and Monte Carlo Results for the Average Frequency Spacing between Maxima," J. Acoust. Soc. Am. 34, 76–80 1962.

[5] K. H. Kuttruff and R. Thiele, Acustica 4, 614 (1954).

[6] M. R. Schroeder, „Improvement of Acoustic-Feeback Stability by Frequency Shifting," J. Acoust. Soc. Am. 36, 1718–1724 1964.

[7] A. D. Pierce, Acoustics ~McGraw-Hill, New York, 1981, Chap. 6, Sec. 6-7.

[8] http://akutek.info/articles_files/stochastics.htm